clear; clc; close all;

p=0.8;n=30;

theta = linspace(0,1,n); lower=zeros(10000,3);upper=zeros(10000,3);

coverage=zeros(10000,3);Intlength=zeros(10000,3);

for i=1:10000

R = binornd(1,p,1,n);

%Frequentist,Using the Wald Interval

mean1=sum(R)/length(R);

lower(i,1) = mean1 - 1.96\*sqrt(mean1\*(1-mean1)/length(R));

upper(i,1) = mean1 + 1.96\*sqrt(mean1\*(1-mean1)/length(R));

%Uniform- Prior Bayesian, it is actually beta(1,1)distribution

[mean2,sigma2]=betastat(1+sum(R),1+length(R)-sum(R));

lower(i,2) = mean2 - 1.96\*sqrt(sigma2/length(R));

upper(i,2) = mean2 + 1.96\*sqrt(sigma2/length(R));

%Beta(8,2)-Prior Bayesian

[mean3,sigma3]=betastat(8+sum(R),2+length(R)-sum(R));

lower(i,3) = mean3 - 1.96\*sqrt(sigma3/length(R));

upper(i,3) = mean3 + 1.96\*sqrt(sigma3/length(R));

for k=1:3

Intlength(i,k)=upper(i,k)-lower(i,k);

if lower(i,k)<=0.8 && upper(i,k)>=0.8

coverage(i,k)=1;

end

end

end

%%%OUTPUT

CoverageFrequentist=sum(coverage(:,1),1)/size(coverage,1)

CoverageUniform=sum(coverage(:,2),1)/size(coverage,1)

CoverageBeta82=sum(coverage(:,3),1)/size(coverage,1)

IntervalLengthFrequentist=mean(Intlength(:,1))

IntervalLengthUniform=mean(Intlength(:,2))

IntervalLengthBeta82=mean(Intlength(:,3))

|  |  |  |  |
| --- | --- | --- | --- |
| N=30 | Frequentist | Uniform | Beta(8,2) |
| Coverage Rate | 0.9447 | 0.3448 | 0.1709 |
| Avg. Interval Length | 0.2785 | 0.0505 | 0.0441 |
| N=5 | Frequentist | Uniform | Beta(8,2) |
| Coverage Rate | 0.6649 | 0.7380 | 0.9398 |
| Avg. Interval Length | 0.5117 | 0.2661 | 0.1724 |

Comments:

1. We can see, when n=30, frequentists has bigger coverage rate but bigger interval length, which means they are mostly distributed around the true p in a big range; Bayesians has smaller coverage rate but smaller interval length, which means they are mostly concentrated in smaller range, but the peak seldom falls near true p.
2. When n=5, compared n=30, frequentists gets both bigger confidence interval and less coverage rate; Bayesians, however, has bigger coverage rate and smaller confidence interval, apparently better than the frequentist prior estimate.
3. In fact, the random Bernoulli generates p=0.5 on average, which redirect the posterior p from 0.8 to 0.5. Under such circumstances, the coverage rate defined in the problem is no longer very meaningful, as the prior estimate changes.